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91. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

There are two unequal square numbers the sum of whose sum, difference, product, and quotient, is a square. Find the two numbers.

Solution by J. H. DRUMMOND, LL. D., Portland, Me.

Any two unequal squares answer the conditions of the question; for, let  $a^2$  and  $b^2$  be the numbers, then  $a^2 + b^2 + a^2 - b^2 + a^2 b^2 + (a^2/b^2)$  must be a square. Reducing, and dropping the factor  $a^2$ , we have  $2 + b^2 + (1/b^2)$ , a square.

This is readily put under the form  $\frac{b^4 + 2b^2 + 1}{b^2}$ , which is the square of  $\frac{b^2 + 1}{b^2}$ .

Q. E. D.

Also solved in a similar manner by G. B. M. ZERR, J. SCHEFFER, and H. S. VANDIVER.

92. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

(a) Find the least three integral numbers such that the difference of every two of them shall be a square number; (b) Find the least three square numbers such that the difference of every two of them shall be a square number.

Solution by J. H. DRUMMOND, LL. D., Portland, Me.

Part 2. Let  $x^2$ ,  $m^2 x^2$ , and  $n^2 x^2$  be the three numbers; then  $m^2 - 1$ ,  $n^2 - 1$ , and  $m^2 - n^2$  must all be squares.

Assume  $m = \frac{p^2 + 1}{p^2 - 1}$  and  $n = \frac{q^2 + 1}{q^2 - 1}$ , and  $m^2 - 1$  and  $n^2 - 1$  will be squares and it remains to make  $\left[\frac{p^2 + 1}{p^2 - 1}\right]^2 - \left[\frac{q^2 + 1}{q^2 - 1}\right]^2 = \square$ , or reducing  $(p^2 q^2 - 1)(\frac{q^2}{p^2} - 1) = \square$ .

This is done by making  $pq = \frac{r^2 + s^2}{2rs}$  and  $\frac{q}{p} = \frac{t^2 + u^2}{2tu}$ .

But  $pq \times q/p = q^2 = \left[\frac{r^2 + s^2}{2rs}\right] \left[\frac{t^2 + u^2}{2tu}\right]$ . Hence  $rs(r^2 + s^2)tu(t^2 + u^2)$  must be a square. Take  $r = f + g$ ,  $s = f - g$ ,  $t = h + k$ , and  $u = h - k$ , and substituting and reducing, we have  $(f^4 - g^4)(h^4 - k^4) = \square$ . Assume  $f^2 = h^4 - k^4 + v^2$  and  $g^2 = h^4 - k^4 - v^2$  and  $(f^4 - g^4) = 4v^2(h^4 - k^4)$ , and  $(f^4 - g^4)(h^4 - k^4)$  becomes  $4v^2(h^4 - k^4)^2$ , a square. Assume  $h^2 = av$  and  $k^2 = bv$ . But  $f^2 - g^2 = 2v^2$ . Assume  $f + g = 4v$  and  $f - g = \frac{1}{2}v$ , then  $f = \frac{9}{4}v$  and  $g = \frac{7}{4}v$ .

Then  $r = 4v$  and  $s = \frac{1}{2}v$ . Then  $pq \left[ = \frac{r^2 + s^2}{2rs} \right] = \frac{81}{16}$ , and  $q/p = \frac{t^2 + u^2}{2tu} = \frac{2(h^2 + k^2)}{2(h^2 - k^2)} = \frac{a + b}{a - b}$ ; and  $h^4 - k^4 + v^2 = f^2 = \frac{81v^2}{16}$ , and  $h^4 - k^4 = \frac{65v^2}{16}$ . Therefore  $a^2 - b^2 = \frac{h^4 - k^4}{v^2} = \frac{65}{16}$ . Now  $pq \times q/p = q^2 = \frac{65(a + b)}{16(a - b)} = \frac{65(a + b)^2}{16(a - b)^2} = (a + b)^2$  and  $q = (a + b)$  and  $p = (a - b)$ . Hence  $m = \frac{(a - b)^2 + 1}{(a - b)^2 - 1}$  and  $n = \frac{(a + b)^2 + 1}{(a + b)^2 - 1}$ , in which  $a$  and  $b$  may be any square numbers which make  $a^2 - b^2 = \frac{65}{16}$ .

It would seem that  $a^2 - b^2 = \frac{65}{16}$  ought to lead readily to a general solution, but  $a$  and  $b$  were both so taken that they must be squares;  $b$  is readily found to